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DAMAGE EVOLUTION AND HETEROGENEITY OF MATERIALS: MODEL BASED ON FUZZY SET THEORY

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Abstract-A mathematical model of damage evolution in heterogeneous materials is developed using the methods of the theory of fuzzy sets. The fuzzy concept of damage is formulated and some applications of this concept are considered. The influence of the material heterogeneity on the damage as well as the heterogenization of the material due to the damage evolution are studied. On the basis of the fuzzy concept of damage, it is shown that the greater the heterogeneity of material, the closer is the material to failure under loading. 1997 Elsevier Science Ltd

NOMENCLATURE

H_h	statistical entropy of the distribution of local strength of heterogeneous material
L	crack length
N_c	amount of components of different strength in the material
$t_f(R)$	time to failure of a body with microcrack density *R*
	membership function of a heterogeneous material into a fuzzy set of material with strength σ
σ	strength of a homogeneous material
σ_i	strength of i-th constituent of a material
σ_L	local stress
$\sigma_{L,cr}$	critical level of the local stress
Y	membership function of a body into a fuzzy set of the behavior of destructed material
Y'	membership function of a body into a complement of the fuzzy set of destructed material
$Y(\mid\sigma)$	conditional membership function of a material (or its constituent) with a given strength into a fuzzy set of the behavior of destroyed materials.

1. INTRODUCTION

INITIALLY. fracture was considered as only a stepwise change of material state-from integrity to the loss of integrity. Then, a number of intermediate states were discovered: first, even a body with large cracks is not always a failed body, the lifetime of a cracked body can be rather long if a crack is less than critical, or if a crack has been arrested. Microcracks in many materials can close after unloading. The initial stages of destruction, namely the initiation and growth of microcracks, can last rather a long time, and differences between states of material at each stage of destruction are rather small. So, the transition from integrity to a failed state is rather smooth: plastic deformation, strain localization, damage initiation, damage evolution, subcritical crack growth, crack arrest, etc. The microfracture appears gradually as well: dislocations movement, local elastic and plastic deformation, accommodation of dislocations, time dependent nucleation of flaw due to the thermal fluctuations[I]. The physical smoothness of the transition from non-failed to failed state of a material is difficult to take into account in the numerical modeling of damage and fracture: although a number of damage models have been incorporated in numerical codes and are used to describe the crack initiation and evolution in different materials [2], the conditions of local failure are often taken arbitrarily (for example, the critical level of the damage parameter by Rice and Tracey was taken as 0.2 [3], or the critical level of Lemaitre's damage parameter was taken as 0.8 [4], without any strong justifications). Therefore, the smoothness of the transition from non-failed to failed state of a material should be modeled in such a way that this model may serve as a basis for the determination of critical conditions of local failure (critical damage factor) and be implemented in the numerical codes which describe the deformation and damage evolution in materials.

Consider the physical properties of a complex material. Usually, the properties of a composite are varied between the properties of all its constituents. Nan [5] has shown that a property of a complex material can present a sum, a product or another combination of properties of its components. The hardness of granite depends on the place and direction of indentation, and on the density of quartz grains in the site of indentation, and can vary in the range of 40% [6]. The uncertainty in local properties of a complex material is determined by the uncertainty of the spatial distribution of its components as well as by the uncertainty of the behavior of a complex system which surely differs from that of the sum of its elements. Thus, the properties of complex materials contain uncertainty.

In order to take into account both the uncertainty and the smoothness of the destruction process as such and the uncertainty of properties and behavior of real materials, it is suggested here to use the theory of fuzzy sets to model the destruction of materials [7, 8]. The transition from non-failed to failed states of a body can be considered as a transition from a fuzzy set of failed state to the complement of this set.

2. CONCEPT OF FUZZY SET IN MODELLING OF STRENGTH OF MATERIAL AND SAFETY OF CONSTRUCTION

The theory of fuzzy sets and fuzzy logic presents methods to describe natural phenomena and their analysis by taking into account the unavoidable uncertainty of the phenomena and our limited ability to describe the phenomena quantitatively. This theory is used in order to convert linguistic variables or other non-precise or subjective appraisals into quantitative and objective data.

Consider some works in which the fuzzy set concept is used in order to study strength, re- liability and other characteristics of materials and products.

Yao [9] considered the problem of damage assessment and suggested using the fuzzy set approach to relate the linguistic and qualitative assessment of damage in structures.

Brown [10] has developed a safety measure, which includes both objective (probability of failure) and subjective (characterized by a membership function, determined with the use of "linguistic wisdom") information about the considered system.

Shiraishi and Furuta [11] analyzed the structural safety taking into account the subjective assessment of engineers with the use of fuzzy probabilities. They expressed a failure event through fuzzy sets, and determined the probability of failure in this case.

Blockley and Baldwin [12] have considered an applicability of the fuzzy logic models in computer knowledge bases. One can see that the probabilistic or statistical methods of analysis are combined with the fuzzy sets in some works.

Klisinski [13] has introduced a fuzzy yield surface which should divide elastic and plastic regions of material behavior. The fuzzy yield surface allows us to model smooth transitions between elastic and plastic states, and to describe the hysteresis loops in a simpler manner than with the use of the two yield surfaces. The cyclic plasticity of materials with and without memory and with and without kinematic and isotropic hardening is modeled with the use of the fuzzy yield surface.

Thus, one can see that the theory of fuzzy sets and fuzzy logic can be efficiently used in order to take into account both uncertainty which is inherent in the natural phenomena as such (here, the uncertainty of yield condition) and that which is caused by a lack of information.

3. FUZZY CONDITION OF FAILURE

In order to describe the process of fracture as not a sudden but rather a smooth transition from one state to another, it is suggested here to introduce a fuzzy definition of the failure condition.

Suppose now that the region of elastic behavior of a body (for simplicity, the loaded body is supposed to be elasto-brittle), which is limited by the condition of local fracture, is a fuzzy

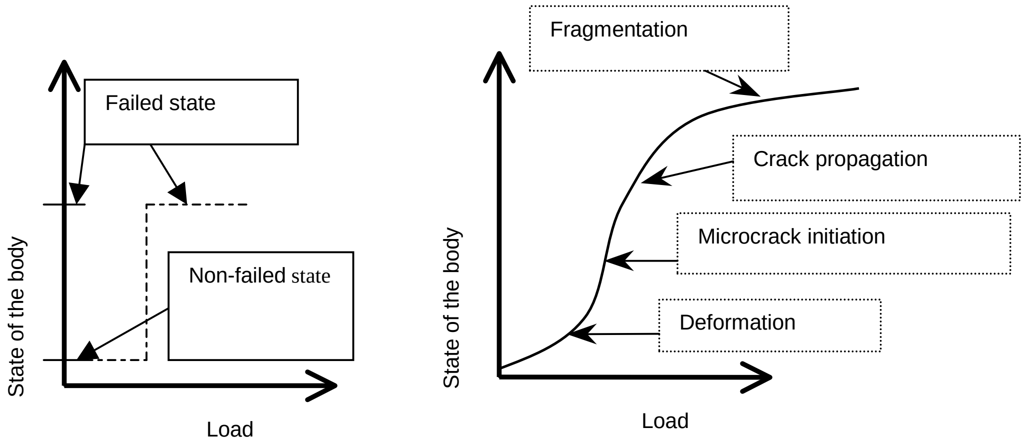

Fig. I. Transition of material from non-failed to failed state: ordinary (a) and fuzzy (b) conditions.

set. This means that a real number Y' on the interval [0;1] can be assigned to each stress level. If the body behaves as a monolithic heterogeneous elastic one, this value is equal to 1. The membership function Y' for this fuzzy set represents the membership degree of the loaded body into the set of behaviors of elastic material. This function Y' is supposed to be continuous [7]. The complement of this set (with a membership function Y = 1 -Y') presents the fuzzy set of behavior of destructed material.

This function characterizes the degree of closeness of the body to failure. Clearly, when the microcrack density in the material increases, the value Y' varies from 1 to 0. (By contrast to the traditional notation in the theory of fuzzy sets, we shall write simply Y, without the symbols of the fuzzy set and elements, the membership degree of which is characterized.) The value Y characterizes the behavior or state of material (failure, or ability to be elastically deformed) and not physical properties of the material.

The fuzzy condition of the transition of material from non-failed to failed state is shown schematically in Fig. 1.

One can note here that the damage parameter in the continuum damage mechanics is usually assigned two meanings: first, microcrack density, and second, the closeness of the material to failure. The conditions of failure are usually formulated as an equality between the damage parameter and some critical value (see for example refs[14, 15]).

Kachanov [16] has formulated two meanings of the damage parameter as follows: "reduction of the effective elastic stiffness" and "the extent of progression towards the final fracture". The latter meaning evidently corresponds to this membership function, which we have just defined. The damage parameter defined in such a way is related to the microcrack density or the deterioration of elastic stiffness in a way not so simple as was supposed, for instance, by Lemaitre[14]. That is why we shall refer hereafter to the "membership function of material state or behavior into the fuzzy set of failed state of material", as a "fuzzy damage parameter".

4. HETEROGENEITY OF MATERIAL AND FUZZY DAMAGE PARAMETER

The fuzziness of damage and fracture is caused among other .factors by the heterogeneity of the material. The local properties of materials are randomly distributed, and any information about them is uncertain.

Consider the properties (in our case strength) of heterogeneous materials. A homogeneous body with strength σ , for example, can be considered as an element of the set of materials of given strength σ with membership degree 1. Yet, if one considers real materials, this set presents a fuzzy set: the strength of material depends on size, local point, conditions of loading, dislocation distribution and movement, etc.; it turns out that the strength of the body is an averaged strength, and this membership degree is less than I. The greater the difference between local properties of material and its total strength, the lower the membership degree of this material into the fuzzy set of materials with this strength.

If a material consists of several components or regions with different properties, the strengths in each are σ_1, σ_2, ..., σ_j, the membership degrees of the material into the fuzzy set of materials with strength σ_1, σ_2, etc. is between 0 and 1. So, one can write the membership function $A(\sigma_i)$, which characterizes the role of each component in the behavior of the material. It can be stated that the function characterizes the closeness of behavior of the material to the homogeneous body, which consists only of this component.

The function $A(\sigma_i)$ describes the heterogeneity of the material, and is analogous to the function Y to some extent: the function Y characterizes the closeness of the state of the body to failure, and the function $A(\sigma_i)$ characterizes the closeness of the behavior of the heterogeneous material to that of the homogeneous one with given properties.

The heterogeneity of material can be characterized also in a more traditional way, i.e. through the statistical entropy of deviations of local properties of the material from averaged properties. This value is related to the membership function A as follows [7]:

$$H_h = (1/N_c) \sum_j \eta_j(\sigma_j) \ln \eta_j(\sigma_j) ,$$

where N_c denotes the amount of components of different strength in the material, j = 1, 2, ..., N_c, H_h the statistical entropy of the local properties of the material, $\eta_j(\sigma_j) = A(\sigma_j)/[\sum_j A(\sigma_j)]$.

Consider the interrelation between the function A and the fuzzy damage parameter. The function Y depends on the strength of material and applied stress (which is taken here to be constant); so, the membership function Y is a conditional one and depends on the strength of material, the variability of which is characterized by the function A. It means that the fuzzy damage parameter can be determined with the use of the formula of conditional membership function[7]:

$$Y = 1- MAX_{\sigma j} (MIN[l -Y(|\sigma_j), A(\sigma_j)]), \qquad (2)$$

where $Y(|\sigma_j)$ is the conditional fuzzy damage parameter for the given strength of material.

To show how to make calculations with this formula, consider the following simple case. The loaded material consists of three constituents. The membership degrees A of each of them are as follows: $A(\sigma_1)$= 0.2, $A(\sigma_2)$== 0.1, $A(\sigma_3)$= 0.6. The fuzzy damage parameters for the materials of each constituent (under given load) are equal to $Y(|\sigma_1)$ = 0.5, $Y(|\sigma_2)$ = 0.15, $Y(|\sigma_3)$ = 0.7. Substituting these values into eq. (2), one can obtain Y = 0.3.

Equation (2) relates the characteristic of variability of material properties and the damaged state in the material.

Yet, not only the strength of the material int1uences the closeness of the material to failure under a given load, but also the degree of destruction of the material int1uences its strength. The growth of damage leads to changes in strength of the material[14]. This dependence can be written as follows:

$$\sigma = f(\sigma_0, Y), \qquad (3)$$

where f is some function, σ_0 is the initial strength of the material at a point. For example, if one uses the strain equivalence principle, we have $f(\sigma_0, Y) = \sigma_0/(1 -R)$.

For this case (i.e. when the local strength of material is changed due to the damage formation), the function $A(\sigma_i)$ depends on the damage distribution. Since this function characterizes variations of local strength from the average strength of the body, it can be taken as a member- ship function of the complement of the fuzzy set of the failed state of the material. One can write

$$A(\sigma_i) = 1 - Y(|\sigma_i) \quad (4)$$

where σ_i is the local strength at the i-th point.

Equations (2) and (4) describe the interrelations between the fuzzy damage parameter and the variability of local material properties from averaged values.

5. FATIGUE DAMAGE GROWTH IN HOMOGENEOUS AND HETEROGENEOUS MATERIALS

Consider now the damage evolution in cyclic loading. When the damaged state of materials is characterized by the density of microcracks, these values are simply summarized in repeated loadings. Yet, here the damaged state of material is characterized by the membership function of the material into the fuzzy set, and this value should be determined in a more complex way.

Let us take a material which consists of several components (minerals, filler and matrix, etc.). The variation of the properties of components from averaged values are described by the function A. Under loading, each of the components becomes damaged and approaches failure to some extent; the degree of closeness of the component to failure is characterized by the function 1 (which depends on both applied load and strength of the component). The damage growth in each of the components also causes further variations of the strength of the component (as described by eq. (4)). This leads to a change in the function A; this membership function is no longer determined only by the initial properties of the material, but also by the weakening of the material due to damage growth. The function A becomes a membership function of a sum of two fuzzy sets, which characterize the variability of local strength of the material from the averaged value caused by initial heterogeneity of the material and damage-induced heterogeneity. Such a membership function is determined as a membership function of a sum of two (or several, in the case of repeated loadings) fuzzy sets:

$$A = 1 - [(1-A_l) + (1-A_2) - (1-A_1)(1-A_2)]$$

$$= A_l A_2 = A_l(l - Y), \quad (5)$$

where A_1 and A_2 are values of the function A, caused by initial heterogeneity of the material and heterogeneity induced by the damage evolution, respectively. In deriving eq. (5) it was taken into account that not the degrees of closeness of the material component properties to the averaged material properties are summarized, but the complements of these fuzzy sets.

In the next loading, the membership function Y depends on the degree of material destruction achieved in the previous loading, as can be seen from formulas (2) and (5).

Equations (2) and (5) describe the approach of the material to failure at repeated loadings. Let us study the damage growth in cyclic loading numerically. Two materials, a homogeneous and a heterogeneous one, are taken. The first one is supposed to consist of 20 components, but the function A for one of the components is equal to 0.999 (this means that the material is practically homogeneous). The fuzzy damage parameter Y for the main component of the material under a given load is taken to be constant and equal to 0.001.

The second material consists of 20 different components again, but the function A for one of these components is 0.7 and for all others 0.05 (this means that the material is relatively heterogeneous). The main component of the material is taken to be the strongest: its damage parameter Y is equal to 0.001. For all other components, Y is equal to 0.03. The changes in $A(\sigma_i)$ due to the damage accumulation were calculated with use of the formulas (4) and (5).

Variations of the fuzzy damage parameter for each time step were calculated by formulas (2) and (5).

Figure 2 shows the fuzzy damage parameter vs. number of loadings for the first and second materials (curves 1 and 2, respectively).

One can see that both curves shown in Fig. 2 look similar, but the damage growth proceeds much more quickly in the heterogeneous material. The fuzzy damage parameter reaches the value 0.6 after nine to ten loadings for the homogeneous material, and after four to five loadings for the heterogeneous material. So, the time to failure in cyclic loading of a heterogeneous

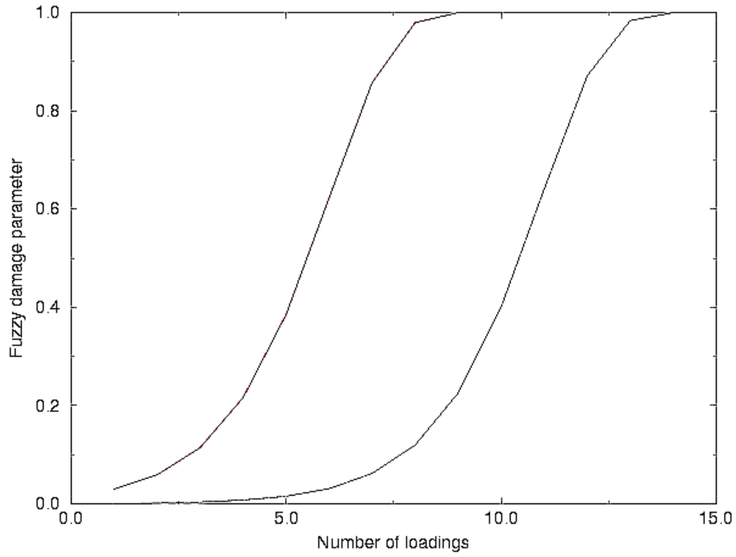

Fig. 2. Fuzzy damage parameter plotted vs. number of loadings for homogeneous (I) and heterogeneous (2) materials.

material is twice as much as that for a homogeneous material. The proportion depends on the degree of heterogeneity of course.

Both curves shown in Fig. 2 have an S shape. Each curve consists of three parts: first, the damage parameter grows almost linearly with the number of loadings, but the rate of growth is rather small; second, the rate of growth of the damage parameter becomes sufficiently greater; and third, the rate of damage growth decreases and approaches zero when the damage parameter approaches one. The transition from first to second stage occurs when Y ~ 0.1 for both materials; the transition from second to third stage occurs at Y about 0.85, independent of the material heterogeneity as well. The first stage of damage evolution corresponds to the independent formation of microcracks in the material. This stage is finished when the microcracks coalesce and form cracks, which begin to grow autocatalytically [17, 18]. Then, the cracks grow with increasing velocity, and correspond to the second stage, which is finished by dividing of the body into parts. The third stage corresponds to the destruction of a partially failed body, up to crushing. The fact that the damage growth rate at the third stage is sufficiently less than at the first or second stage is confirmed by the experimental observation that the energy consumption in crushing of a material is much more than at the formation of initial cracks [19]. The energy needed for crack formation due to the damage coalescence is greater than the specific energy of crack growth as well; this theoretical result obtained on the basis of the fractal model of fracture presented by Mishnaevsky [18] corresponds to our conclusion that the damage growth rate at the initial stage of destruction is much less than at the second stage.

6. HETEROGENIZATION OF THE MATERIAL DUE TO THE DAMAGE ACCUMULATION

Consider now interrelations between the degree of heterogeneity of a material and its closeness to failure.

Let us take 100 different materials with different distributions of properties and study the effect of the heterogeneity on the fuzzy damage parameter. All materials are loaded with the same force. Each of them consists of 20 components with different properties (strength σ_i). The components of the materials are characterized by the values of $A(\sigma_i)$ (i.e. the degree of variation of the strength of this component from the average strength of the material) and $Y(|\sigma_i)$ (i.e. the fuzzy damage parameter for this component of the material). The statistical entropy of the local strength distribution was calculated in each case by the formula (1), and the fuzzy damage

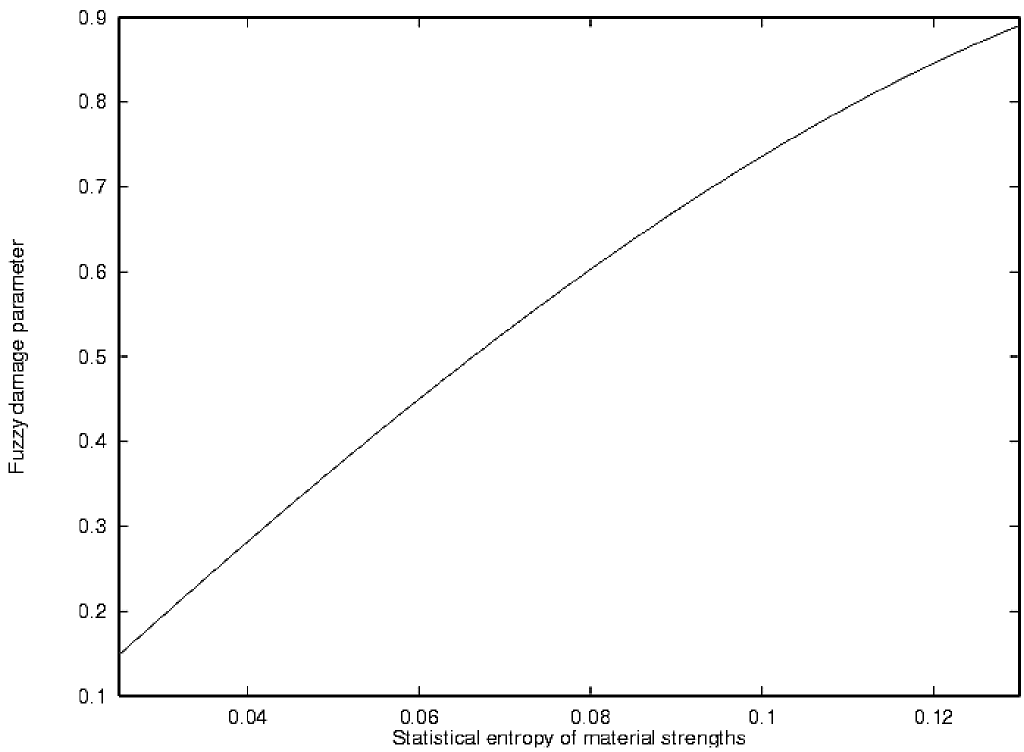

Fig. 3. Fuzzy damage parameter plotted vs. statistical entropy of local strength of material.

parameter by the formula (2). In order to ensure the different entropies for each material and non-homogeneous distributions of the components in each material, the following function $A(\sigma_i)$ was used:

$$A(\sigma_i) = (1/M_n)j^{\,0.2rm}. \quad (6)$$

where r_m is the number of the considered material, r_m = 1, 2, ..., 100, j is the number of the considered component of this material, j = 1, 2, ..., 20, M_n a constant of norma1ization. The values of Y for each component were calculated as random numbers between 0 and 1.

Figure 3 shows the fuzzy damage parameter of a material vs. the heterogeneity of the material, which is characterized by the statistical entropy of distribution of local strengths of the material.

One can see from Fig. 3 that the more heterogeneous the loaded material, the closer the material to the failed state. The dependence is monotone. One should note that this conclusion was to be expected: it is known that the firmest materials are homogeneous monocrystals; the large difference in properties of the filler and matrix in the metal matrix composites can lead to the formation of damage and damage growth in the matrix.

This result can also be compared with the conclusions from the analysis of Fig. 2: the greater the heterogeneity of the material, the closer it is to failure.

Consider now the reverse effect: the influence of the approach of a material to failure on its heterogeneity. Figure 4 shows the statistical entropy of the material strengths plotted vs. the fuzzy damage parameter. The statistical entropy for each step was calculated with the use of eqs (1), (4) and (5). One can see that the greater the damage parameter the more heterogeneous the material. Physically, it means that high-damaged and low-damaged regions as well as cracks and crack systems are formed in the loaded body during damage evolution and the strengths of different regions of the material become more different due to different levels of damage in these regions.

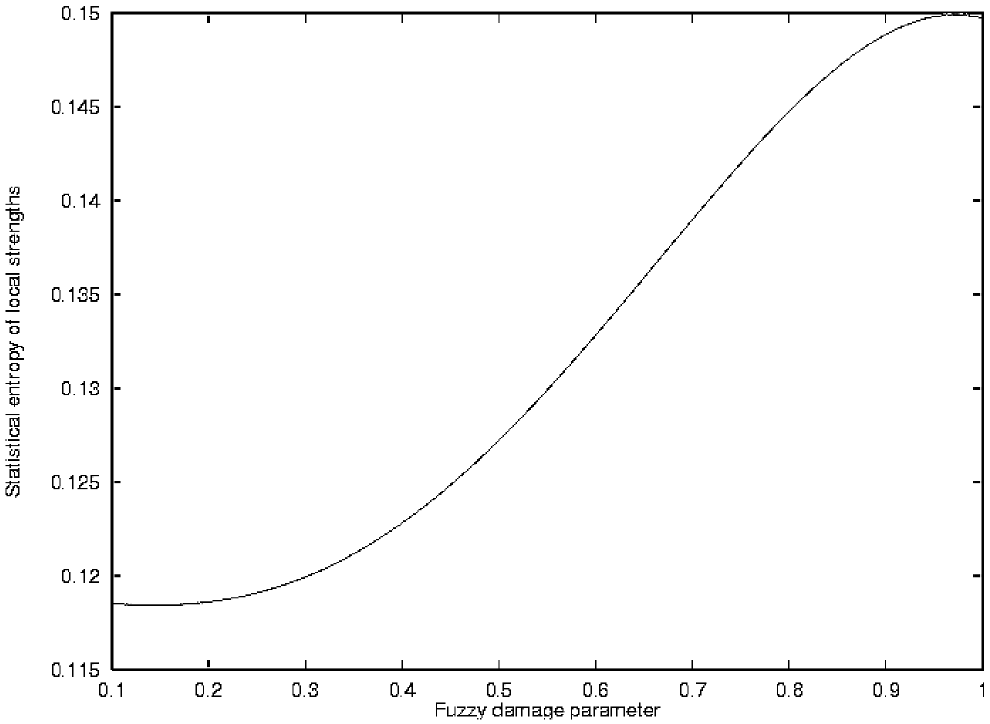

Fig. 4. Statistical entropy of local strength of material plotted vs. the fuzzy damage parameter: heterogenization of a material.

One can compare this conclusion with the experimental data on rock fragmentation in indentation described in ref. [19]. It is known that the strained volume of rock is divided into several zones during damage accumulation: cracked zone, zone of crushed (powdered) rock, zone of distributed microcracks, etc. Such formation of differently damaged zones precedes the removal of rock chips in all processes of mechanical destruction. Thus, the experimental observations of the mechanism of rock fragmentation confirm the theoretical conclusion that the damage growth leads to the heterogenization of the local properties of the material. It is of interest to compare this conclusion with the results of Carpinteri and Yang [20]: the authors have shown, on the basis of the analysis of the evolution of the fractal dimension of a micro- crack net, that a strained material becomes more heterogeneous during the damage evolution.

7. STRESS DEPENDENCE OF FUZZY DAMAGE PARAMETER

One can see that the fuzzy damage parameter is not defined as a function of stress in the framework of this model. It reflects the fact that the transition from non-failed to failed state of a material is taken here as an uncertain one, and the fuzziness of destruction may be determined by a number of factors of different nature, both objective and subjective.

In order to establish some interrelations between the parameter and local stress, consider the definition and sources of fuzziness of fracture in more detail. The fuzziness of the transition of a material from one state to another may be caused by the fuzziness of the destruction process as such, by the uncertainty of definitions (may a body be considered as destroyed when it does not bear some load, or when it is divided by a crack into two parts, or when it becomes powdered?), by the heterogeneity of the material (a local stress which is failing at one point of a material may be not failing at another point), or by the graduality of the transition (i.e. the material may seem to be firm, whereas its microcrack density increases and approaches the critical one).

If the fuzziness of destruction is considered as an inherent feature of the destruction, the fuzzy damage parameter can be determined on the basis of subjective appraisals.

The definitions of degrees of destruction of a material can also be introduced artificially: for example, a volume may be considered as failed (and the membership function Y equal to some critical value Y_{cr} when the defects in this volume form an infinite cluster and the body is divided into parts.

If the fuzziness of destruction is caused by heterogeneity of the material, it means that the fuzzy set of the failed state of the material presents an ordinary set for homogeneous materials and the membership function Y for ideally homogeneous materials can be presented as a step function:

$$Y_0=\{\begin{matrix} 1, \sigma_L > \sigma_{L,cr} \\ 0, \sigma_L < \sigma_{L,cr} \end{matrix} \quad (7)$$

where σ_L is the local stress, $\sigma_{L,cr}$ is the critical stress. The formula (7) is based on the above definition of a failed body; the value $\sigma_{L,cr}$ is measured as a stress at which the body is divided into parts by an as-formed crack. Then, the step function Y_0 is substituted into eq. (2) and the fuzzy damage parameter for the heterogeneous material can be calculated as well. One should note that for homogeneous materials eq. (2) reduces to an identity. As a generalization of formula (7), one can take Zhurkov's condition of microfracture [21]; in this case, the formula (7) is written in the following way:

$$Y_0=\{\begin{matrix} 1, t > \Delta t(\sigma_L) \\ 0, t < \Delta t(\sigma_L) \end{matrix} \quad (8)$$

where t is the time during which the local stress exceeds some value σ_L, $\Delta t(\sigma L)$ is the critical value of the time t, at which the volume fails and which is calculated by the formula.

If the fuzziness of destruction is determined by the graduality of the transition from integrity to the failed state of the material, the membership function of the material into the fuzzy set of failed states can be related to temporal characteristics of the destruction process. The strained volume goes from the state with Y = O to the state with Y_{cr} during a time calculated from the formulas of refs [17, 18]. So, the membership function Y may be defined as the time required for the material at a given stage of damage evolution to be failed divided by the summary duration of the process of transition from non-failed to failed state:

$$Y = t_f(R) / t_f(0), \quad (9)$$

where $t_f(R)$ is the time to failure of a body with microcrack density R. The formulas for tf(R) are given in ref.[18].

8. CRACK GROWTH IN HETEROGENEOUS MATERIAL

Consider now the crack propagation. A crack in a loaded body induces a stress field which is described by a formula like

$$\sigma_T = \sigma_n \ f(\theta) \sqrt{L/2r}\ , \quad (10)$$

where σ_T is the component of the stress tensor, L is the crack length, r the distance between a point and the crack tip, θ the angle between the line r and the plane in which the crack lies, f(θ) a given function of θ and σ_n the normal stress acting on the specimen.

As discussed in refs [1, 17,18], the propagation of a crack proceeds as follows: one or several microcracks are formed in front of the tip of a large crack, stress fields from the crack and microcracks interact, and then finally some volume in front of the crack tip is failed and the crack is increased by a new surface formed in this volume. So, the damage evolution and destruction in the small volume in front of the crack tip determine the crack propagation.

Let us consider a small volume of material in front of the crack. The fuzzy damage parameter in the area in front of the crack is changed from a value equal to the averaged fuzzy damage parameter throughout the volume (or simply equal to 0) to the value $Y = Y_{cr}$.

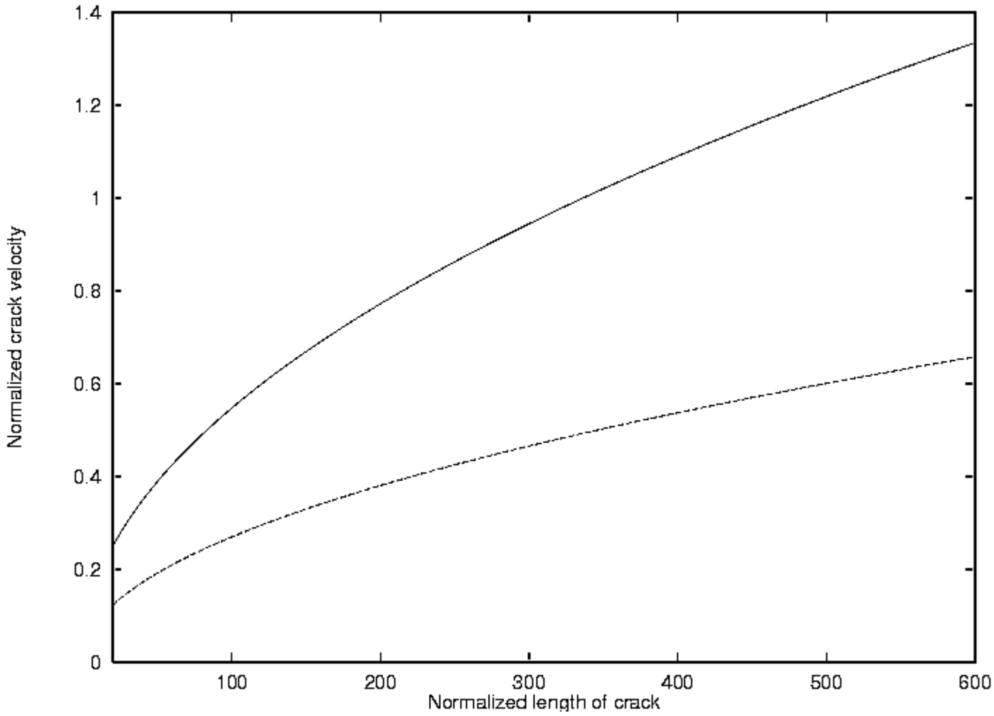

Fig. 5. Crack velocity plotted vs. crack length: (I) homogeneous and (2) heterogeneous materials.

Assuming that the fuzziness of destruction is caused by the heterogeneity of the material, one can study the effect of material heterogeneity on the crack growth. One may also establish an interrelation between the fuzzy model of damage evolution and the model of crack growth based on the kinetic theory of microfracture[17]: if one accepts the condition (8) and considers a homogeneous material, one obtains a crack velocity vs. crack length relation similar to the formula from ref.[17]:

$$\frac{dL}{dt} \propto \exp(\text{const}\sqrt{L}) \qquad (11)$$

Consider now the crack growth for both homogeneous and heterogeneous materials, using the analysis of the influence of the material heterogeneity on the damage evolution given in previous paragraphs. The crack growth in two materials, with the same parameters as considered above, is simulated. The value Y_{cr} is taken to be 0.3. The fuzzy damage parameters of each constituent of the material are considered for some point a distance r from the crack tip, in which the local stress is equal to some given value. The velocity of crack growth was calculated as the distance r divided by the time (in time steps) which is necessary for destruction of the volume with linear size r. The following input data are used: r = 1, fractal dimension of the crack D = 1, applied normal stress σ_n = I, initial length of the crack 20, $\sigma_{L,cr}$ = 25000 for the stronger component in both materials and 8340 for the weaker component. The values of A for these two components are 0.999 and 0.7 for homogeneous and heterogeneous materials respectively.

The crack growth velocity (in 1/time step) plotted vs. crack length is presented in Fig. 5.

One can see that the crack velocity is much greater in the heterogeneous material than in the homogeneous one, at the same length of crack.

9. FUZZY AND STATISTICAL CONCEPTS OF FRACTURE

It is of interest also to compare the developed approach with the statistical concepts of strength.

The fuzzy damage parameter Y characterizes not the possibility of transition from one state to another (failed state of material), but the present state of the material, which can be inter- mediate between these two states. It is a fact that the state of the body, which is characterized by its membership degree into the fuzzy set of corresponding state, is not changed due to duplication, repeating of the same elements, or increase in size of the body [7]. Yet, the possibility of transition of the body from one state to another depends on the duplication of elements or the size of the body. The difference between some results of the probabilistic and reliability models and the developed model lies in this fact. One can see that the characteristics of the present state of a body (for example Young's modulus) do not depend on the duplication, number of elements, or size of the body, whereas the characteristics of the possibility of change of state under corresponding conditions (for example time to fracture or failing load) are determined to a larger extent by the number of duplicating elements.

Kauffmann [7] has also mentioned that one can define a probability function on fuzzy sets. Such a definition was suggested by Shiraishi and Furuta [11]. If the condition of transition of the material from a non-failed state to a failed one is given not as a sharp boundary (like an equality between applied and critical loads) but as a membership function of the material into the fuzzy set of the failed state (like above), the probability of the transition is equal to an expectation of the membership function:

$$\mathrm{Prob(Failure)} = \int_{-\infty}^{\infty} \mathrm{Y(z)p(z)dz}$$

where z is a parameter of the system (elements of the fuzzy set in the discrete case), p(z) is the probability distribution of z, Y(z) is the membership function of an element z into the fuzzy set of the failed state.

Such a definition presents a generalization of the concept of probability of failure on uncertain events.

Then, consider the scale effect on the strength of bodies. In the statistical theory of strength, it is assumed that a body is failed when the number of defects in it reaches some critical level; the scale effect (i.e. the fact that the greater a body the less its specific strength) is explained by using the assumption that the greater the body the more defects it contains[22].

These assumptions cannot be accepted in the fuzzy concept of strength of damaged bodies: as was said above, the state of a system from independent elements with the same membership function does not depend on the number of elements connected in parallel [7] (in contrast to the reliability or probability of failure of this system). So, the scale effect cannot be explained in the framework of the fuzzy concept of strength in the same way as was done with the statistical theory of strength.

Yet, one can put forward another explanation of the scale effect, which seems to also be justified physically. As shown above, the fuzzy damage parameter increases with increasing heterogeneity of the material. For the range of sizes in which the scale effect can be observed, the increase of size means most often an increase in the heterogeneity of the material. Any material has some heterogeneous structure at several levels (dislocation structure at microlevel, grain boundaries, grain clusters in particulate composites, stratified structure in rocks at macrolevel, etc.), and the greater the size of the specimen used, the more levels of structure influence the strength of the material and form its heterogeneity[23]. So, the greater the specimen, the greater its heterogeneity Hh and, consequently, the greater its closeness to failure under loading.

10. CONCLUSIONS

The influence of material heterogeneity (i.e. the extent of the variability of properties of material components from the averaged material properties) on the damage evolution was studied with use of the theory of fuzzy sets.

With use of this model, it was shown that:

- the damage growth in heterogeneous materials under cyclic loading proceeds much more intensively than in homogeneous materials;

- the greater the statistical entropy of strengths of material components, the less the lifetime of a product from this material.

It is shown as well that the heterogenization of a loaded material proceeds during damage accumulation and evolution. The closer the loaded material to fracture, the more heterogeneous it is.

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