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METHODS OF THE THEORY OF COMPLEX SYSTEMS IN MODELLING OF FRACTURE: A BRIEF REVIEW

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Abstract - Theoretical investigations of damage and fracture of materials which are based on the concepts of the theory of complex systems are reviewed and analyzed. The models of fracture, which have been developed with the use of the methods of following theories, are considered: theory of phase transitions and statistical physics, percolation and fractals theories, theories of dynamical systems, bifurcations and self-organization. The main achievements, perspectives and limitations of the application of these methods in modelling of fracture are analyzed.

1. INTRODUCTION

Fracture is a complex process which proceeds simultaneously at several levels (micro-, meso- and macrolevels and these levels of the process interact; the random processes at each level (i.e. thermofluctuations, heterogeneity of the material, etc) have an important bearing on the fracture; such phenomena as the kinetic stability, fractality and, in some cases, self-organization can be observed in fracture as well. Processes of this sort can be modeled with the use of the mathematical methods of synergetics, as such, and the theories which constitute the basis of the synergetics (or can be considered as its parts), i.e. the theory of phase transitions, dynamical systems, fractals, information theory, etc [1]. These theories taken together are called here the theory of complex systems [1], since the term „synergetics“ can be understood as the theory of self-organization only. We use here the terms „complex system“ and „theory of complex systems“ in the sense, in which they have been used by Nicolis and Prigogine in [1].

Traditionally, the models of destruction describe only one level of process: the fracture mechanics describes macrofracture; the continuum damage mechanics models the behaviour of microcracked bodies only. It is known that the models of complex processes which are based on the theory of complex systems allow to obtain in many cases some new results which can not be obtained with the use of traditional methods or their combinations. Here, we consider and review the models of fracture which are based on the theory of complex systems. This paper seeks to collect the main results obtained in this area during last years, to determine the perspective directions of the investigations as well as the weak points of the models.

The simplest way to model a complex system with unknown structure is to use the probabilistic or energetical (i.e. based on the energy balance) methods. The well-known theory of Griffith has been obtained just on the basis of the consideration of the energy balance in crack growth. The probabilistic models of failure are presented in the works of Weibull [3], Freudenthal [4] etc. In the statistical theory of strength, it is supposed that the loaded body fails when the (random) amount of microdefects in the body exceeds some critical value. The other direction is the application of the reliability theory [5,6]. The probability of failure is supposed to be equal to the exponential function of applied load.

Within the last years the many new data on the physics and mechanics of fracture have been obtained; in the same time, the new discipline called „synergetics“ has been developed. The rather evident analogy between many of objects of the synergetics and fracture (distributed microcracks and ensembles of particles; crack and nuclei of new phase; cracks and viscous fingers,etc) suggested a number of researchers to use the methods of synergetics and related disciplines to model fracture. Many of these investigations are considered here. The reviewed papers were divided into groups according to both the main parts of synergetics (for example, the theory of bifurcations, theory of phase transitions) and the traditional directions in modelling of fracture (for instance, the kinetic theory of strength, fractal models of fracture). Although the theories of fractals and percolation can not be considered as integral parts of the synergetics, we included in our review investigations based on these theories for the following reasons: the percolation theory and the fractal growth models are close related with the phase transitions theory; the fractality is characteristic of many complex and self- organizing systems and objects; the fractality of complex objects is concerned with their structure, conditions of formation and behaviour [1].

2. METHODS OF PHASE TRANSITION THEORY AND STATISTICAL PHYSICS

The thermodynamical theory of phase transitions can be considered as a basis of the theory of self-organization [2]. The model of fluctuationally induced phase transformation is used to describe the formation of dissipative structures [1,2]. Such models are applied to describe damage and fracture as well. The analogy between phase transitions (first of all, melting) and fracture has been noted rather long time ago [7,8]. Now the investigations in this direction are conducted rather intensively.

There are several aspects of the analogy betwen fracture and phase transitions: physical similarity (both fracture and melting are caused by breaking links between elements of body; fracture can be considered as melting along some planes, fluctuational mechanism of fracture and phase transitions etc); similarity of macrobehaviour (a crack can be considered as a nucleus of a new phase, stepwise change of the free energy of system in fracture as well as in phase transitions, etc) and similarity (but not identity) of micromechanisms (formation, growth and coalescence of nuclei of fracture or new phase; crack growth proceeds similarly to the diffusion-controlled aggregation of particles, etc). There are also several peculiarities of fracture as compared with the phase transition: for example, fracture is always irreversible; although (dissipative) structures are formed in some cases, the entropy of system decreases in fracture; shapes and sizes of nuclei of a new phase (i.e. cracks) influence on the process of their growth (here, one means the stress concentrantion in the vicinity of a crack tip); etc.

The models of fracture based on the phase transitions theory can be divided into two main groups: first, determination of the thermodynamical potential of the system and its application to study the behaviour of cracked body, and the second, based on the fluctuational models of phase transitions.

Consider some investigations in which the thermodynamical potential of damaged/cracked body is determined and then applied to predict the material or cracks behaviour.

Jaeger and Englman [9] suggested the thermodynamical theory of fracture in heterogeneous solids. They formulated the thermodynamical potential of free energy, which constitutes a functional of the probability distribution of microcracks density, and includes the terms of elastic surface energy, correlation in the activation of the microcracks, influence of the available cracks on the elasticity of material, crack-crack interaction and entropy term. Then, this theory is applied to the uniform medium approximated by a discrete lattice. This model is used to study the effect of disorder and the influence of grains size on the ductility of material.

Brady [10] used the Landau-Ginzburg theory to study rock fracture. He supposed that the order parameter is the shear stress. The brittle fracture is modeled as a system which consists on three phase, i.e. solid material, damaged one and macrocracks. The condition of fracture is determined with the use of critical relation in the vicinity of the tricritical point.

Watanabe [11] has used the Landau-Ginzburg potential as well, however, for the other purposes: he modeled not the behaviour of cracked body, but the crack propagation under dynamical loading. He has taken the normalized energy release rate in crack growth as the order parameter. Taking into account the experimental relation for the energy release rate, the author has shown that the stochastic differential equation for change in the energy release rate in crack growth closely resembles the equation for laser radiation.

Ostoja-Starzewski [12] has applied the methods of the random fields theory to study the damage formation, plastification and brittle cracking in materials with random multiscale effects. He defined the nearest neighbor Gibbs potential and informational entropy of disorder, and determined the internal energy as a function of the damage state in the material. The constitutive law for the damaged material was derived.

Naimark, and Naimark and Silbershmidt [13,14] considered the formation of crack from the distributed microcracks as a kinetic phase transition in the ensemble of defects. They determined the change in the free energy in loaded body which is a function of the damage tensor. This model allows to describe the microcracks distribution in impact loading, creep, etc. Popov, Ivanova and Terentyev [15] considered the kinetics of defects ensembles by the analogy with the kinetic theory of gases.

Then, consider the investigations which are based on the assumption about thermofluctuational mechanism of fracture.

Yokobori [16, 17] has developed a theory of fracture nuclei, in which the crack velocity is determined with the use of the Arrhenius equation. The cracks (i.e. the nuclei of fracture) are taken as penny-shaped or spherical growing objects. Contrary to the classical theory of phase transitions, the formation of nuclei of new phase (i.e. microcracks) occurs not in random points, but in the places of high stress concentration. The critical crack size is determined from the condition of the minimum of Gibbs free energy.

Zhurkov [18] has obtained experimentally the time-to-fracture versus applied load and temperature relation, which looks like the formula for the time interval between fluctuations in the theory of fluctuations. On the basis of this analogy, Regel et al [19] and Hsiao [20] have interpreted the Zhurkov's formula as follows: the thermofluctuations lead to the breakage of interatomic bonds, and the accumulation of the broken bonds leads to fracture; the applied stress changes only the potential barrier.

Hsiao [20] has generalized the Zhurkov‘s model for the case of high and low stress beyond the region of applicability of Zhurkov‘s results. Grabar [21] has shown that the models of Zhurkov and Griffith are interrelated, and have determined a crack rate limit on the basis of the Zhurkov‘s theory.

Krausz and Krausz [22] considered the microcracks initiation due to the dislocations accumulation, and obtained the formula for the crack growth velocity which looks similar to the Yokobori‘s formula.

Consider some other models which are based on the concepts of thermodynamics and statistical physics as well, but can not be included in above groups.

Chudnovski [23] noted that fracture can be considered as a „ partial“ phase transition (melting): the melting consists on two stages - breakage of atomic bonds and mixing of particles; fracture is just breakage of the atomic bonds along some planes. Chudnovski developed also the entropical criterion of local fracture (the entropy growth density).

Olemskoy and Naumov [24] considered applicability of the criterium of melting energy to determine the local strength of materials, and derived a formula for strength on the basis of this approach. Frantziskonis [25] considered the displacements gradient of micro-medium as the random field. It was shown that there is an analogy between the statistical description of displacements and Mindlin‘s microstructural theory. The internal length (in the icrostructural theory) is analogous to the correlation length in the statistical one.

Thus, one can draw the following conclusions. Contrary to the traditional methods, the methods based on the phase transitions theory and statistical physics allow to take into account the time- and temperature- dependence, non-linearity and stochastic nature of fracture; they make it possible to describe the behaviour of both cracked body and cracks as such; they allow for the structure of material and micromechanisms of fracture.Yet, the problem of determination of the thermodynamical potential of cracked and damaged body is rather complex and can not be considered as solved.

The assumption about the fluctuationally induced fracture of macrobodies is not justified experimentally; fracture of macrobodies is caused by the processes at mesolevel (in particular, by the coalescence of microcracks), not at atomic level [26]. The interrelation between the breakage of atomic bonds due to the fluctuations, and fracture of materials is to be studied.

3. METHODS OF THE PERCOLATION AND FRACTAL THEORIES

The analogy between the geometrical phase transition (percolation) and the crack formation due to the microcracks coalescence is rather evident (at least, for the case of two- dimensional percolation).

The fractality of surfaces of natural objects (for instance, the well-known example of the length of a coast [27]) has been noted also among the first fractals. Fractality is observed usually in very complex systems, which are described with the use of non-linear differential equations [1]. The percolation clusters also constitute one of classes of the fractal objects. Investigations devoted to the fractality of fracture can be divided into three groups: models based on the percolation theory; simulation of the crack growth on the lattice models; theoretical investigations of fractal crack growth. The experimental works and investigations of the size effect on strength are not considered here

The percolation theory approach has been used by Chelidze, and Chelidze and Gueguen [28,29], Krajcinovic and Basista [30], Sahimi and Goddard [31], Delaplace et al [32], Nan [33] etc. Chelidze [28] considered the fracture crack as an infinite percolation cluster. He used the lattice model of percolation, and supposed that the interaction between neighbouring defects depends on the local stress. Chelidze and Gueguen [29] have shown that the surface energy is proportional to the correlation length of the fractal network to the power D-3, where D - the fractal dimension of the cracks network. Delaplace, Pijaudier-Cabot and Roux [32] have developed numerical (lattice) and analytical models of damage evolution and failure, and have shown that the percolation and screened percolation models agree with numerical results only for initial stage of damage evolution or a very disordered medium. On the basis of the developed „random damage model“, which describes all stages of damage evolution, they determined conditions, at which an initial stage of damage evolution, controlled by the variability of material properties, gives way to the next stage, influenced by the current redistribution. Nan [33] has used the percolation and fractal theories in order to model elastic properties of composites and to relate the microstructure and properties of composites.

Ostoja-Starzewski [12] considered failure of a body as the percolation of damage, i.e. the stochastic cooperative field phenomenon. He determined the probability of percolation as a function of the distribution of microcracks and microfracture probability. He compared also the Weibull‘s statistical theory of strength and the percolation model of fracture.

Mishnaevsky Jr [34, 35] used the percolation model of fracture caused by the damage coalescence to determine the fractal dimension of initial cracks and the probability distribution of their sizes. It was shown that the surface roughness of initial part of a crack and the specific surface energy needed to form a crack by the mechanism of microcracks coalescence are much more than these for the propagating crack.

There is a number of investigations of the fractality of fracture which are based on the computer simulation of fracture on the lattice models. In this models microfracture is taken as a breakage of bonds between points. For example, Takayasu [36] used the square lattice from brittle bars, and studied the space and time scaling of the amount of broken bars. It was shown that the crack growth is self- similar in time.

Louis and Guinea [37,38] modeled fracture on the triangular network from springs. The elastoplastic behaviour of elements is taken into account as well. The authors have shown that the fractal dimension of crack depends on the strength of damaged elements (springs).

Herrman [39] has formulated the problem on growth of micro- and macrocracks as the moving boundary problem. The medium is modelled as a set of points embedded in a grid. To study the influence of material heterogeneity on fracture, the quenched disorder was introduced. It as shown that the influence of the disorder on fracture is very strong, and the scaling laws in the breaking characteristics exist.

Termonia and Meakin [40] have considered the two-dimensional square network and used the kinetic theory of strength to determine the probability of the bond breakage. This model allowed to determine the fractal dimension of a crack, which is about 1.27.

Williford [41] considered the self-similarity of microcracks in the vicinity of a crack tip, and obtained the fractal relation between the specific surface energy, and macrocrack length and its growth increment.

Then, consider some analytical investigations of the fractality of fracture.

Lung [42] proposed the fractal model of intergranular brittle fracture, and has shown that the surface energy of fracture depends on the fractal dimension of crack, and, then, on the grain size in metals.

Xie Heping [43] considered the fractality of cracks, caused by multiple crack branching, and has obtained a relation between the fractal dimension of crack and angle of branching. On the basis of the energy balance equation, he deduced a relation between the crack extension rate and the fractal dimension of a crack.

Mishnaevsky Jr [44] has used the analogy between the diffusion- controlled aggregation and crack growth due to the random microcracks joining, and determined the fractal dimension of crack (which is about 1.3). The crack velocity versus crack length relation which allows for the fractality of fracture was deduced.

Krausz [45] applied the random walk theory to study the crack propagation. It is clear that this approach makes it possible to determine the fractal dimension of crack as well.

On the basis of this analysis, one can draw the following conclusions. The fractality of crack is caused by the randomness of the initiation of microcracks which form the crack [28-30, 34], crack branching [43], conditions of crack growth [44], granularity and structure of material [42], self-similarity of the microcracks formation [41]. The fractal dimension of a crack can be determined on the basis of the computer simulation [36-40], fractal growth model [44] or branching model [43], with the use of interrelations between the critical indices in the percolation theory [34, 35]. One can see some intersections between the fractal models, and the phase transitions and the statistical physics models. Except for the investigations, which are based on the methods of both the theory of fractals and statistical physics [12, 39,40], one can refer to the percolation theory models which are related with the theory of critical phenomena and phase transitions, or to the consideration of the crack path as an attractor [46], what is related with the theory of fractals as well.

4. BIFURCATIONS, STABILITY AND SELF-ORGANIZATION IN FRACTURE

The theory of dynamical systems, and, in particular, the theory of bifurcations and stability, are applicable to all developing systems. Yet, in order to use the methods of these theories, differential equations describing the system are required. In most cases, the differential equations for the behaviour of loaded cracked body are unknown. That is why these theories are of limited usefulness in modelling of fracture. Nevertheless, some researchers used the concepts of theses theories in their investigations.

The question about applicability of the self-organization concept to describe fracture calls for further analysis. On the one hand, fracture is irreversible process, which leads to the degradation and destruction of material, and raises the entropy of system. On the other hand, one can observe in fracture such phenomena as kinetic stability, formation of dissipative structures etc, which are characteristic of the self-organization.

Consider some investigations in this area. The stability of crack under different conditions of loading has been investigated by many researchers with the use of the methods of fracture mechanics [47].The bifurcation of a running crack (i.e. the crack branching) is considered as an instability phenomenon as well [26, 43]. Guz and Nazarenko [48] used the concept of the local instability near defects to study the fracture of materials in compression along cracks.

Dyskin and Muehlhaus [49] have studied crack interaction in crack array using the dislocation approximation, and established equilibrium bifurcations in the behaviour of a cracked body.

On the basis of his model of the kinetic phase transition in the ensemble of defects, Naimark [13,14] has shown that the variety of forms of failure and plasticity is determined by the effect of self-organization under the nucleation of collective modes for the damage tensor.

Belyaev, and Belyaev and Naimark [50, 51] considered the damage localization in metals and ceramics under impact loading. It was shown that the self-organization in the microcracks distribution can be observed, and a new type of dissipative structures (blow-up structures) is formed. The conditions of formation and kinetics of development of the dissipative structures are studied.

Kaski et al [52] simulated the influence of disorder and dissipation in materials on the fracture behaviour. The elastic interaction strength of bonds in lattice model was described with the Born Hamiltonian. The scaling behaviour, variations of crack tip velocity, crack arrest and branching and other phenomena have been considered. It was shown that strong dissipation makes a material ductile and can lead to a crack arrest.

Andrianopoulos and Kourkoulis [53] have developed a unified approach to the crack path instability phenomena. Both directional and velocity instabilities are considered. The authors introduced the twin-crack model, which is based on the assumption that the macroscopical instability is the result of the microscopic multibranching at the crack tip. They determined the stress intensity factor, dynamic stress field and other parameters of fracture.

Vujocevic et al [54] studied the strain localization in solids under compression with the use of a model of a disordered medium as an ensemble of individual particles bonded together. The evolution of correlation length and the onset of strain localization in disordered media was investigated theoretically.

Pook [46] has applied the concepts of the chaos theory to the fatigue crack path, and has shown that the concept of attractor helps in qualitative description of fatigue crack growth. The formation of mode I branch crack is considered as chaotic event, and the crack path is a bifurcation. The stability of a crack is analyzed as well. It was shown that the minor variations have a significant effect on the direction of crack growth.

Bazant and Jirasek [55] have used a nonlocal model of continuum damage, developed by Bazant [56] and based on micromechanics of crack interaction in order to study an evolution of interacting growing microcracks and localization of cracking damage into an infinite planar band. They analyzed bifurcations of load-displacement diagram during an incremental loading process.

Walraef and Aifantis [57] studied formation of different dislocation patterns (cellullar, labyrint, etc) in loaded material.

Ivanova [58] developed a synergetical model of fatigue fracture of metals which is based on the following assumptions: fatigue fracture is determined by cooperative behaviour of two competitive mechanisms of microfracture, i.e. microshear and microbreakage, each of them depends on critical density of dislocations or disclinations, respectively; cracked state of a body is similar to melting; the local state and stability of material depend on a ratio between shear modulus multiplied by critical elastic energy and Young modulus of the material multiplied by enthalpy of material. Depending on the parameter, local shear or breakage, which correspond to rotational or translational local instabilities, can occur.

Panin, and Panin et al [59, 60] have modeled destruction of materials, as cooperative hierarchical process of damage evolution, which is followed by competitive processes of accumulation and dissipation of energy. On the basis of kinetic equation of state and energetical condition of local fracture, they developed a mathematical model of material destruction. They have shown that the self-organization of structure of a loaded material, which leads to redistribution of accumulated energy in the material and can be observed as fragmentation, polygonization, etc. in the material, takes place.

Gradov and Popov [61] modeled damage accumulation in materials as many-level process, taking into account the interaction between defects. They presented free energy potential as a function of microcracks density and simulated change of phase states in damaged bodies.

Marigo and Daguerre [62] studied the damage evolution in brittle materials and have shown that the stability requirement induces the development of microstructure in material.

Shanyavsky [63] has modelled the fatigue crack growth in aircraft components with the use of the concept of synergetics. The hierarchy of self- organization processes in fatigue crack growth was considered in relation with the kinetic diagram. Bolotin [64] considered the crack growth as the instability phenomenon. The cracked body is treated as a mechanical system with unilateral nonholonomic constraints. The conditions of stability of fracture, sub- equilibrium and non-equilibrium states of this system are analyzed.

Pyrz [65] investigated the dispersion of fibres in fibre-reinforced composites on the basis of second-order properties of spatial point patters created by fibre centroids. He determined a second-oder mark intensity function which characterizes disordered arrangement of fibres in a composite, and developed a discrete model of evolution of microcracks and formation of percolating crack, which proves the availability of scale effect in some classes of fibres dispersion. The influence of the geometrical disorder of inclusions arrangement in real matetrial on fracture was considered also by Pyrz and Bochenek [66].

Michlashevich and Chigarev [67] modelled the crack growth on the basis of the assumption that the crack direction corresponds to the minimum of the functional of work. The authors have shown that the stochastization of crack growth direction occurs at some conditions.

Gulluoglu and Hartley [68] simulated the evolution of dislocations microstructure in metals with the use of the molecular dymanics method. It was shown that the stable structures of dislocations are formed in loaded material. The formation of the structures is considered as the self-organization.

Germanovich et al [69] have shown that the crack formation can be modeled with the use of the catastrophe theory.

Watanabe [11] has deduced the stochastic differential equation for the change in the energy release rate in crack propagation, and has shown that the oscillations of crack velocity occur in dynamic fracture.

Ivanova and Grabar [70] have demonstrated that the cascade of bifurcations can be observed in fatigue crack growth. On the basis of the authomodelity condition, they obtained an equation which describes the kinetics of fatigue crack growth. It was shown that the kinetics of growth of fatigue crack at Paris‘ stage is a critical phenomenon of the class of Landau.

Mishnaevsky Jr, and Mishnaevsky Jr and Schmauder [71, 72] modeled the damage localization in heterogeneous material. The wave propagation and damage evolution in loaded material have been modelled with the use of stochastic differential equations. The authors have shown that the localization of damage and formation of highly damaged zones are most intensive at initial stages of damage evolution.

Thus, one can see that the crack behaviour as well as the behaviour of damaged bodies demonstrate such phenomena as bifurcations (crack branching; loss of stiffness of damaged body), catastrophes, instability at some stages (authocatalytic growth, local instability), oscillations, kinetic phase transitions, stochastization, availability of attractors, etc. These phenomena are difficult to model in the context of the fracture mechanics. The best plan to be followed is to combine the methods and results of the fracture mechanics and continuum damage mechanics with the concepts and methods of synergetics.

5. CONCLUSIONS

The presented review allows to determine the main achievements, perspectives and weak points of the considered approaches.

The application of the theory of phase transitions makes it possible to describe the behaviour of both the cracks and cracked body; to describe the crack and microcracks evolution in the context of unified approach; to take into account the influence of the random factors and disorder on fracture. The fluctuational odels make it possible to study the time and temperature dependence of fracture, to allow for the atomistic mechanisms and andomness of fracture. The application of these methods allows us to use the mathematical apparatus of corresponding theories, which includes models of many non-linear and complex phenomena.

Yet, there are several weak points as well: the classical theories of phase transitions are based on the assumption that the nuclei of new phase are formed in random points whereas microcracks in loaded body are formed in the places of high stress concentration; the analogy between crack and nucleus of new phase is limited, and the limitations of this analogy are to be studied. The justification of the kinetic theory of strength which is based on the thermofluctuational model of bonds breakage does not take into account the available data about the physical mechanisms of macrofracture (which is caused by the microcrack coalescence, not by the atomic bonds breakage). One should note also that the model of equilibrium fluctuations is not applicable to the dynamic fracture.

The percolation theory as well as the fractal growth theory allow evidently to relate macro- and microfracture, taking into account the physical mechanisms of fracture. Yet, one should note that the applicability of the percolation model in fracture is limited: only formation of macrocracks from randomly distributed microcracks can be modeled as percolation; when the stress field of a large crack begins to influence on the microcracks initiation, the percolation theory in its classical form is inapplicable. Besides, the generalization of the percolation models of fracture on 3-dimensional case presents some additional difficulties [73].

Contrary to the other models which have both merits and demerits as compared with the fracture mechanics, the fractality of crack is experimentally and theoretically proved fact. It seems to be useful to apply some results of the fractal theory of fracture also in models which are based on the fracture mechanics approach.

In fracture, the phenomena which are characteristic of the self-organization proceed: kinetic phase transitions, formation of dissipative structures,bifurcations, stochastization, etc. With the use of the synergetical approaches, the following problem can be solved: the determination and simulation of conditions of damage localization, and modelling of non-stable or complex regimes of crack propagation. One can suppose that there are many other problems in the theory of fracture, where the concepts of synergetics can be applied: for example, trajectory of growing crack in the heterogeneous and/or micro- cracked material, crack arrest due to the crack- microcrack interaction, etc. Yet, in order to use the methods of synergetics, it is necessary to have physically justified differential equations, which describe the evolution of cracks and microcracks, and behaviour of cracked body.

One should note here that despite the fact that the considered approaches are interrelated in the context of synergetics (for example, phase transitions, fluctuations and stability, fluctuations and bifurcations, attractors and fractality, informational models and stability, etc), the models of fracture based on these methods do not take into account these interrelations. It seems to be very promising to stimulate the development of fracture models, based on the synergetical methods, by taking into account the results of interrelated areas of the synergetics. For example, the stability of crack propagation and microcracks evolution, as well as the conditions of the self-organization can be studied with the use of the fluctuational models of fracture; the fractal dimension of crack depends on the regimes of crack propagation (i.e. stochastization, stable or non-stable regime, etc.)

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